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NUMERICAL INVESTIGATION OF THE HEAT EXCHANGE AND FIRING OF REACTIVE CHANNEL WALLS BY A HIGH-TEMPERATURE SWIRLING-GAS FLOW

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A study is made of the process of firing of reactive channel walls by a swirling flow of hot gases. It is shown that the swirl of the flow leads to a reduction in the time of establishment of thermal equilibrium and that of firing; these times are reduced in a stepwise manner with the appearance of a recirculation zone in the strong-swirling flow.

Introduction. The process of firing is modeled with allowance for the following assumptions:

(1) constant values of the velocity, pressure, and density of the gas are maintained at entry into the channel at $t > 0$;

(2) a nonreactive firing gas is considered; the gas flow is axisymmetric;

(3) in modeling the process of firing, we use a solid-phase model [1] within whose framework it is assumed that the totally exothermic processes in the condensed phase of the firing are responsible for it.

Mathematical Model. To describe the gas dynamics we have used the physicomathematical model presented in [2, 3] in detail. This model includes:

(a) two-dimensional unsteady Navier–Stokes equations [4]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{1}{r}\frac{\partial \rho v r}{\partial x} = 0 , \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{1}{r}\frac{\partial \rho u v r}{\partial x} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\mu_{\text{eff}}\left(2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial v r}{\partial r}\right)\right)\right] + \frac{1}{r}\frac{\partial}{\partial r}\left[\mu_{\text{eff}} r\left(2\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)\right], \tag{2}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho u v}{\partial x} + \frac{1}{r}\frac{\partial \rho v^2 r}{\partial x} = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial x}\left[\mu_{\text{eff}}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial r}\right)\right] +$$

$$+ \frac{1}{r}\frac{\partial}{\partial r}\left[\mu_{\text{eff}} r\left(2\frac{\partial v}{\partial r} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial v r}{\partial r}\right)\right)\right] - 2\frac{\mu_{\text{eff}} v}{r^2} + \frac{\rho w^2}{r}, \tag{3}$$

$$\frac{\partial \rho w}{\partial t} + \frac{\partial \rho u w}{\partial x} + \frac{1}{r}\frac{\partial \rho v w r}{\partial x} = \frac{\partial}{\partial x}\left[\mu_{\text{eff}}\frac{\partial w}{\partial x}\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{\mu_{\text{eff}}}{\sigma_{r\varphi}} r^3 \frac{\partial}{\partial r}\left(\frac{w}{r}\right)\right] - \frac{\rho v w}{r}; \tag{4}$$

(b) modification of the k–ε turbulence model that allows for the anisotropy of turbulence and the influence of centrifugal forces on the processes of generation and dissipation of turbulence [5]:

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$$\frac{\partial \rho k}{\partial t}+\frac{\partial \rho u k}{\partial x}+\frac{1}{r}\frac{\partial \rho v k r}{\partial x}=\frac{\partial}{\partial x}\left[\frac{\mu_{\text{eff}}}{\sigma_k}\frac{\partial k}{\partial x}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\mu_{\text{eff}}}{\sigma_k}r\frac{\partial k}{\partial r}\right]+G_k-\rho\varepsilon\,, \tag{5}$$

$$\frac{\partial \rho \varepsilon}{\partial t}+\frac{\partial \rho u \varepsilon}{\partial x}+\frac{1}{r}\frac{\partial \rho v \varepsilon r}{\partial x}=\frac{\partial}{\partial x}\left[\frac{\mu_{\text{eff}}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\mu_{\text{eff}}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial r}\right]+(C_1-G_2\rho\varepsilon)\frac{\varepsilon}{k}\,. \tag{6}$$

In dependences (5) and (6), G_k is the dissipative function, which is calculated as

$$G_k=\mu_{\text{t}}\left\{2\left[\left(\frac{\partial u}{\partial r}\right)^2+\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{v}{r}\right)^2\right]+\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2+\left(r\frac{\partial w/r}{\partial r}\right)^2\right\}\,. \tag{7}$$

The values of the constants are selected in accordance with the recommendations of [5]: C_1 = 1.44, C_2 = 1.92(1 – C_3 Ri), C_μ = 0.09, σ_k = 1, and σ_ε = 1.3. The Richardson number Ri is determined by the following expression:

$$\text{Ri}=\frac{k}{\varepsilon^2}\frac{w^2}{r}\frac{\partial\,(wr)}{\partial r}\,. \tag{8}$$

The effective viscosity (μ_{eff}) is determined as the sum of the molecular (μ) and turbulent viscosity (μ_{t}). The turbulent viscosity can be calculated with the use of the k–ε turbulence model [5]:

$$\mu_{\text{t}}=C_\mu\rho k^2\varepsilon^{-1}\,, \tag{9}$$

where C_μ = 0.09 is the constant of the turbulence model.

To describe heat exchange in the gas flow we use the equation of convective heat exchange [6]

$$c_p\left(\frac{\partial \rho T}{\partial t}+\frac{\partial \rho u T}{\partial x}+\frac{1}{r}\frac{\partial \rho v r T}{\partial r}\right)=\frac{\partial}{\partial x}\left[\lambda_{\text{eff}}\frac{\partial T}{\partial x}\right]+\frac{1}{r}\frac{\partial}{\partial r}\left[\lambda_{\text{eff}}r\frac{\partial T}{\partial r}\right]. \tag{10}$$

The heat transfer and the chemical reaction in the solid phase $r_{\text{int}} < r \le r_{\text{ext}}$ are described by the equations of heat conduction and chemical kinetics with allowance for the exothermic reaction following the Arrhenius law:

$$\rho_{\text{w}}c_{\text{w}}\frac{\partial T_{\text{w}}}{\partial t}=\frac{\partial}{\partial x}\left(\lambda_{\text{w}}\frac{\partial T_{\text{w}}}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{\text{w}}\frac{\partial T_{\text{w}}}{\partial r}\right)+Qz_0\,(1-\eta)^n\exp\left(-\frac{E}{RT_{\text{w}}}\right), \tag{11}$$

$$\frac{\partial \eta}{\partial t}=z_0\,(1-\eta)^n\exp\left(-\frac{E}{RT_{\text{w}}}\right). \tag{12}$$

The thermophysical and kinetic characteristics have been taken from [1]. The initial data for the wall material are as follows: E = 1.46·10^2 kJ/mole, Q = 1.13·10^6 J/kg, z_0 = 0.9·10^{14} 1/sec, c_{w} = 1.46 kJ/(kg·K), λ_{w} = 2.34·10^{-2} J/(m·sec·K), and ρ_{w} = 1.6·10^3 kg/m^3. We considered a nitrogen flow as the firing medium. An equation of state relating the medium's density to the temperature and pressure is added to the system of equations given above. For ideal gases this equation has the form $p = \rho RT$. The specific heat of the gas c_p is taken to be temperature-independent. The estimate Pr_{eff} = 0.7 is used for the effective Prandtl number $\text{Pr}_{\text{eff}} = c_p\mu_{\text{eff}}/\lambda_{\text{eff}}$.

As the initial conditions for determination of the aerodynamics of the flow we used the steady-state solution of the problem on gas flow in a cylindrical channel with the inlet temperature and the wall temperature equal to T_0. Due to the ellipticity of the system of differential equations, to close the problem we must set up boundary conditions at all the boundaries of the computational domain.

1. The inlet boundary conditions are determined for all the variables. We prescribe the distribution of the flow velocity and take the inlet kinetic energy of turbulence to be in proportion to the kinetic energy of the average flow:

$$x = 0: \quad u = u_{\text{in}}, \quad v = 0, \quad w = u_{\text{in}} \tan(\varphi), \quad k = \text{Tu} w_{\text{in}}^2, \quad \varepsilon = k_{\text{in}}^{3/2}/(\beta r_{\text{int}}).$$

Here β = 0.05 and Tu = 0.03 are the constants of the model, r_{int} is the radius of the channel, and φ is the angle of swirl of the flow.

2. On the channel axis, we specify traditional conditions:

$$r = 0: \quad \frac{\partial u}{\partial r} = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial k}{\partial r} = 0, \quad \frac{\partial \varepsilon}{\partial r} = 0.$$

3. The sticking condition holds on the channel walls. To determine the turbulent characteristics we assume the local equilibrium in the wall region:

$$r = r_{\text{int}}: \quad u = 0, \quad v = 0, \quad w = 0, \quad k_{\text{n.w}} = \frac{\tau_{\text{w}}}{\rho \sqrt{C_\mu}}, \quad \varepsilon_{\text{n.w}} = \frac{k_{\text{n.w}}^{3/2} C_\mu^{3/4}}{\kappa r_{\text{n.w}}}.$$

Here κ = 0.4 is the von Kármán constant, τ_{w} is the stress on the wall, and $r_{\text{n.w}}$ is the distance from the wall to the nearest wall node.

4. At exit, the axial components of the gradient of tangential velocity and of the turbulent characteristics k and ε are assumed to be equal to zero. The values of the radial velocity v in the outlet cross sections are taken to be equal to zero:

$$x = L: \quad \frac{\partial u}{\partial x} = 0, \quad v = 0, \quad \frac{\partial w}{\partial x} = 0, \quad \frac{\partial k}{\partial x} = 0, \quad \frac{\partial \varepsilon}{\partial x} = 0.$$

To close the thermochemical part of the problem we set up the following initial and boundary conditions:

(1) at the initial instant of time, we prescribed the gas temperature and the wall temperature; the chemical-transformation depth η was set equal to zero:

$$t = 0: \quad T = T_0, \quad T_{\text{w}} = T_0, \quad \eta = 0;$$

(2) the conditions at the boundaries at $t > 0$ can be formulated in the following manner: the gas temperature was prescribed at entry and the end of the channel wall was considered to be heat-insulated:

$$x = 0: \quad T = T_{\text{in}} > T_0, \quad \frac{\partial T_{\text{w}}}{\partial x} = 0;$$

(3) At exit from the channel, we modeled the conditions of thermal stabilization of the flow and heat insulation of the wall:

$$x = L: \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial T_{\text{w}}}{\partial x} = 0;$$

(4) on the flow axis, we set up the symmetry conditions:

$$r = 0: \quad \frac{\partial T}{\partial r} = 0;$$

(5) the conditions of conjugate heat exchange were formulated at the gas–solid wall boundary:

$$r = r_{\text{int}}: \quad T = T_{\text{w}}, \quad \lambda_{\text{w}} \frac{\partial T_{\text{w}}}{\partial x} = \lambda_0 \frac{\partial T}{\partial x};$$

(6) at the external boundary of the channel wall, we specified the conditions of heat exchange with the environment:

$$r = r_{\text{ext}}: \quad \lambda_{\text{w}} \frac{\partial T_{\text{w}}}{\partial x} = -\alpha (T_{\text{w}} - T_{\text{env}}),$$

where T_{env} is the environment temperature.

In the course of the investigation, we varied the following parameters: the wall thickness, the initial flow temperature, the wall temperature, the environment temperature, and the heat-transfer coefficients.

Procedure of Solution. The system of differential equations (1)–(2) was solved numerically with the use of the finite-volume method [7] in accordance with which the finite-difference equations are obtained by integration of the differential equations over control volumes containing the nodes of the finite-difference grid.

Numerical solution was carried out with the use of a staggered grid; the nodes for the axial and radial velocity components were arranged at the center of the sides of control volumes for scalar quantities. Computations were carried out on the grid with 210 nodes in the axial direction and 176 nodes in the radial one. A series of calculations on sequences of clustering grids was performed to evaluate the computation accuracy. The results of testing have shown that a twofold reduction in the step of the base grid, on which the basic calculations were carried out, along the axial and radial coordinates leads to a change of no more than 3% in the values of the basic variables.

In approximating the convective terms, we used a QUICK upstream scheme (proposed by Leonard [8]) of third order; the diffusion terms were approximated by the central-difference scheme. The continuity equation was satisfied indirectly with the use of the SIMPLEC procedure [9].

The system of nonlinear algebraic equations resulting from the approximation was solved numerically with the use of iterations. The iterative convergence was considered to be attained if the standard deviation was no higher than 1% for all the variables.

Analysis of the Results. Anticipating consideration of the thermochemical part of the problem, we consider certain features of the structure of a swirling flow in a cylindrical channel.

The swirl of the gas flow leads to the appearance of the tangential velocity component w and the formation of the field of centrifugal forces proportional to $\rho w^2/r$ and intensifying the gas motion in the radial direction. In the axial region, when the swirl intensity is low ($\varphi = \arctan(w/u) < 50^{\circ}$), the pressure gradient caused by the presence of centrifugal forces and insufficient for the appearance of a return-flow zone leads only to a decrease in the u values in the axial region. When the swirl in the vicinity of the axis is larger, a higher rarefaction occurs, which initiates the central zone of return flows (Fig. 1). Deceleration of the flow in the axial flow region or even its reversal at large φ leads to an increase in u at the periphery. The $u(r)$ diagram has its maximum for a certain $r = r_{\text{m}}$, with its further smoothing as x grows. As φ increases, we observe an increase in the maximum value of u and a shift of r_{m} to the wall.

Restructuring of the flow associated with the formation of the recirculation zone on the initial portion of flow and attenuation of the swirl due to viscosity with distance from the inlet leads to the radial motion of the substance; the larger the α, the higher the values of the radial velocity. Thus, at $\varphi > 50^{\circ}$, the values of v are more than an order of magnitude higher than the value of the radial velocity at $\varphi = 0$.

Consequently, convective transfer of momentum, a substance, and heat in the radial direction becomes substantial in the swirling flow.

To determine the time and temperature of firing by a high-temperature supersonic jet we used the critical-condition method of Averson, Barzykin, and Merzhanov [1]; this method records the instant of ignition of a condensed substance by the coincidence of the heat fluxes from hot gases and due to the chemical reactions occurring in decomposition of the k substance.

Intensification of heat transfer in the radial direction, as the swirl grows, leads to an intensified heat exchange and an increase in the heat flux from the gas into the wall. Figure 2 shows the change in the dimensionless heat-trans-

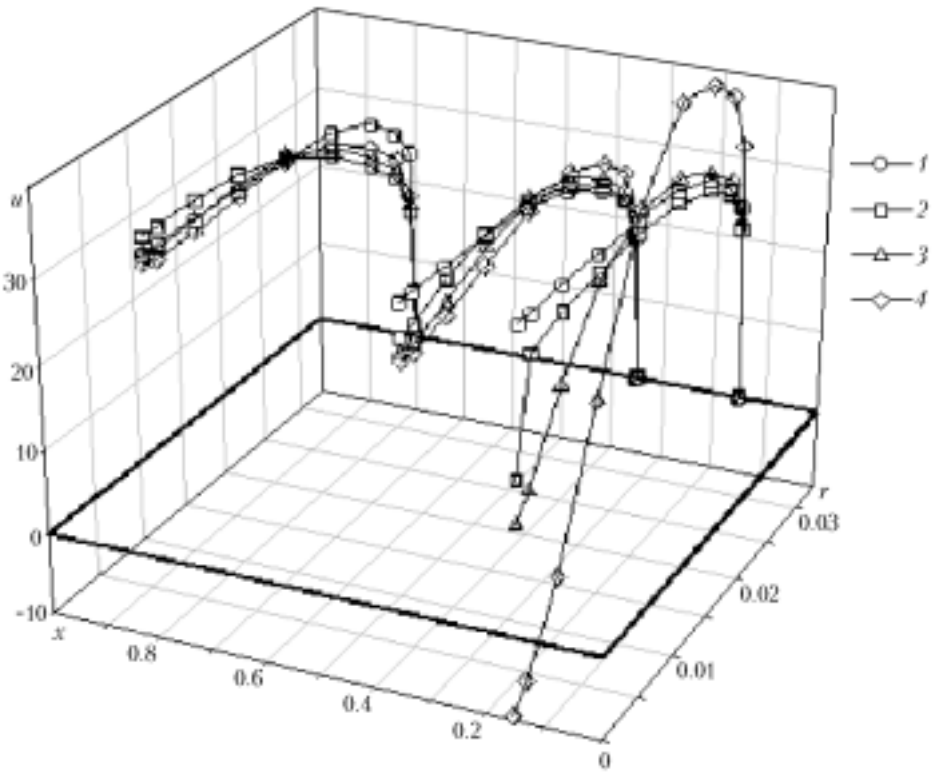

Fig. 1. Radial distribution of the axial velocity in the channel: 1) φ = 0; 2) 40^o; 3) 60^o; 4) 80^o. u_{in} = 30 m/sec. *u*, m/sec; *x*, *r*, m.

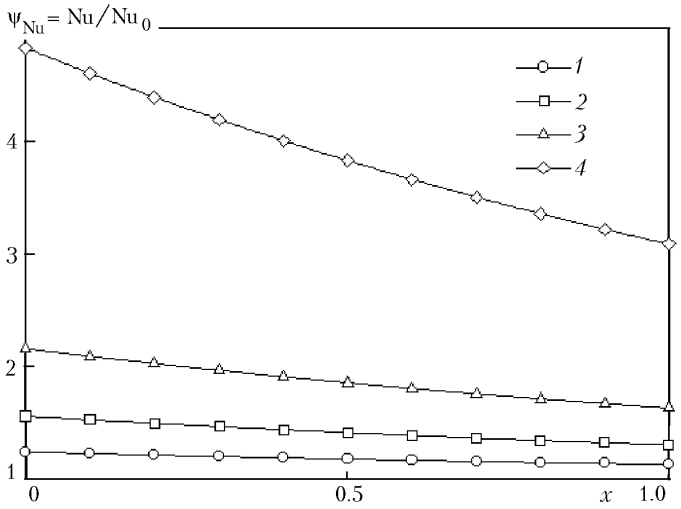

Fig. 2. Change in the dimensionless heat-transfer coefficient $\psi_{Nu} = Nu/Nu_{\varphi=0}$ downstream: 1) φ = 20^o; 2) 40^o; 3) 60^o; 4) 80^o. Fo = $0.2 \cdot 10^{-3}$. *x*, m.

fer coefficient $\psi_{Nu} = Nu_{\varphi\neq0}/Nu_{\varphi=0}$ characterizing the influence of the swirl on the change in the heat-transfer coefficient, which is calculated for Fo = $0.2 \cdot 10^{-3}$. As is clear from the figure, we have an increase in Nu in the entire flow region as the swirl grows. The increase in the heat transfer of the flux to the wall is most pronounced at $\varphi > 60^o$, when the return-flow zone has been formed in the flow core and the wall is intensely washed by the hot-gas flow. The largest heat transfer is on the initial portion of flow, when centrifugal forces dominate viscous ones. Downstream, with degeneration of the swirl, Nu tends to values obtained for forward flows. We note that the increase in u_{in} (or in the Reynolds number $Re = 2\rho u_{in} r_{in} \mu^{-1}$ in a more general case) leads to a growth in heat transfer.

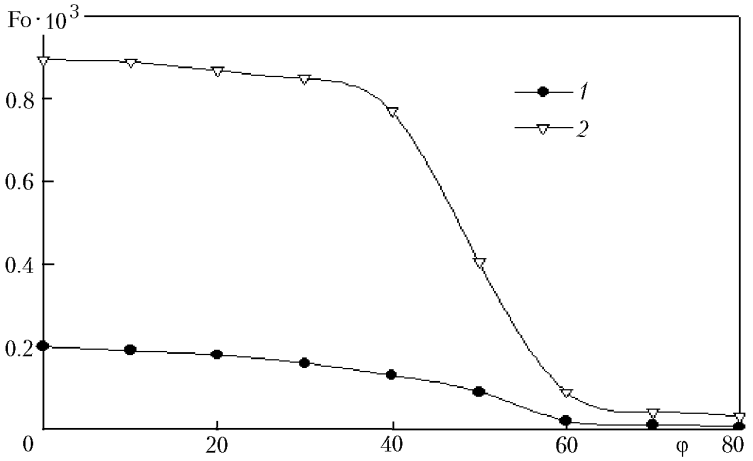

Fig. 3. Dimensionless times of the processes of establishment of thermal equilibrium (1) and firing time (2) for different values of the parameter of swirl φ. φ, deg.

Thus, the time of heating of the wall to the hot-gas temperature is substantially reduced. Intensification of the course of the chemical reaction in the channel wall results, which leads to an increase in the wall temperature, and the time of firing of the channel walls decreases in the case of a high-energy reaction.

Figure 3 gives the dimensionless times of the processes of establishment of thermal equilibrium Fo_{th} and firing Fo_* for different values of the parameter of swirl φ. As is clear from the figure, the characteristic times Fo_{th} and Fo_* decrease as the swirl intensity increases. Their change is very slight at $\varphi < 20^o$, which is attributed to the slight influence of the swirl on the aerodynamics of the flow. At $20^o < \varphi < 60^o$, the influence of the swirl on the flow structure and consequently on the conditions of heat exchange with the wall and the firing time becomes more substantial. At $\varphi > 60^o$, we observe a stepwise decrease in Fo_{th} and Fo_*. The reason is that the recirculation zone occurs and heat transfer into the wall increases multiply.

CONCLUSIONS

In the course of the investigations carried out, we have studied the mechanism of firing of reactive channel walls by a swirling hot-gas flow. It has been shown that the swirl of the flow leads to a reduction in the time of establishment of thermal equilibrium and in the firing time. These times are reduced in a stepwise manner with the appearance of a recirculation zone in the strong-swirling flow.

NOTATION

C_1, C_2, and C_μ, constants in the turbulence model; c_w, specific heat of the channel walls, kJ/(kg·K); c_p, specific heat of the gas, kJ/(kg·K); E, activation energy, kJ/mole; Fo, Fourier number; G_k, dissipative function, W/m^3; k, turbulent kinetic energy, m^2/sec^2; L, channel length, m; n, order of the reaction; Nu, Nusselt number; p, gas pressure, Pa; Pr, Prandtl number; Q, thermal effect of the chemical reaction, kJ/kg; R, universal gas constant, J/(kmole·K); r, radial coordinate, m; Re, Reynolds number; Ri, Richardson number; T, temperature, K; t, time, sec; Tu, parameter of turbulence; u, longitudinal velocity component, m/sec; v, radial velocity component, m/sec; w, tangential velocity component, m/sec; x, axial coordinate, m; z_0, preexponent, 1/sec; α, heat-transfer coefficient, W/(m^2·K); β, constant in the turbulence model; η, depth of decomposition of the wall material; ε, turbulent-energy dissipation, m^2/sec^3; κ, Kármán constant; λ, thermal conductivity, W/(m·K); μ, coefficient of dynamic viscosity, kg/(m·sec); ρ, density, kg/m^3; σ_k, $\sigma_{r\varphi}$, and σ_ε, constants in the turbulence model; τ, stress, J/m^3; φ, angle of swirl of the flow, deg; ψ_{Nu}, dimensionless heat-transfer coefficient. Subscripts: eff, effective; env, environment; ext, exterior wall; in, inlet parameters; int, interior wall; m, maximum; n.w, wall (near-wall); th, thermal equilibrium; t, turbulent; w, wall; 0, initial; *, firing.

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